Structural Modeling of Posttensioned Members

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Abstract: This state-of-the-art paper offers a comprehensive overview of the modeling techniques used for the analysis of post-tensioned structures. The merits and limitations of each of the modeling schemes are discussed within a consistent and comparative framework. Several numerical examples are used to illustrate the tacit features of the models. The balanced loading, primary actions, hyperstatic (secondary) actions, and prestressing moment concepts commonly used in the analysis of prestressed structures are revisited and clarified. The work concludes with an example of the most recent modeling technology - the discrete modeling of tendons. The example illustrates the calculation of long-term prestress losses as an integral part of the analysis as opposed to the traditional approach where long-term losses are computed independently from the solution.

INTRODUCTION

Prestressing is used to control crack formation in concrete, reduce deflections, and add strength to the prestressed member. Prestressing thus plays a significant role in the structural integrity and desired response of the member.

The authenticity and relevance of the analysis of a prestressed concrete member rests, first and foremost, on appropriate modeling of the tendons. Several different modeling schemes are used to represent prestressing tendons, each of which has some degree of approximation [Scordelis 1984]. This paper provides a brief description of each scheme and places their comparative features in perspective.

The focus of the paper is on the post-tensioning tendons. These are groups of prestressing strands, wires, or rods, which are stressed against the concrete member after the concrete is set. The tendons are typically given a profile in the vertical plane to enhance their load resisting characteristics.

The contribution of a tendon to the response of the prestressed member depends on the stress in the tendon at both service and strength limit conditions, the tendon's profile and its cross-sectional area. There has been little difficulty in representing the tendon profile accurately in structural analysis. The challenge facing the various modeling schemes has been the accurate determination and representation of the stress in the tendon, including immediate and long-term effects. Depending on the structure, the validity of the overall analysis may depend upon the inclusion of such effects into the model.

The most critical considerations in the structural modeling of post-tensioned tendons are:

Immediate Loss of Stress in Tendon

Fig.1-a shows a post-tensioned tendon within a partially displayed concrete member. When the tendon is pulled with a force F_o at the stressing end, it will elongate.

The elongation will be resisted by friction between the strand and its sheathing or duct, however. As a result of this friction, there will be a drop in the force in the tendon with distance from the jacking end. The friction is composed of two effects: curvature friction which is a function of the tendon's profile, and wobble friction which is the result of minor horizontal or vertical deviations from the specified profile.

After they are stressed, the tendons are typically anchored with conical wedges. The strand retracts upon release and pulls the wedges into the housing of the anchorage device; this forces the wedges together and locks the strand in place. The retraction of the tendon results in an additional stress loss over a short length of the tendon at the stressing end. This loss is illustrated by the difference between the jacking force and the final force at the left end of the force profile in Fig. 1-b.

Fig. 1-b shows the variation in force along the tendon. In general, the stress loss will depend on the tendon's length, its profile, its friction characteristics, the design of its locking mechanism and the stressing force. The combined loss due to all of these effects is commonly referred to as the *friction and seating loss.* The force profile will be similar for a tendon stressed at two ends; although there will be a seating loss at each end the total loss due to friction will be less.

Elastic Shortening

Most prestressed members are reinforced with several tendons which are stressed and anchored one after another. As each tendon is stressed, the compression in the concrete member increases. The elastic shortening of the concrete due to the increase in compressive stress causes a loss of prestressing force in tendons which were previously stressed and anchored. The stress loss in each tendon will depend on the total number of tendons in the concrete member and the sequence of stressing among other factors.

Long-Term Losses

Long-term losses cause a reduction in tendon stress with time. These losses are due to several factors:

- Relaxation of the prestressing steel: Prestressing tendons lose a fraction of their initial stress with time due to the metallurgical characteristics of the prestressing material.

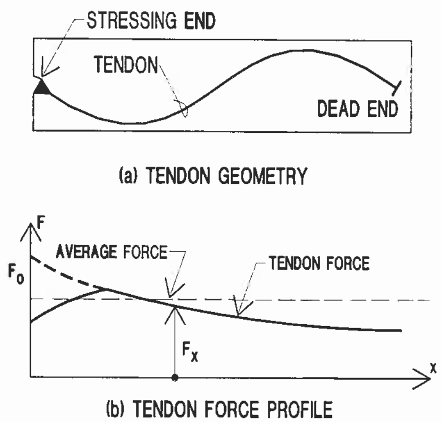

FIG. 1. Loss of Prestressing due to Friction and Seating

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The loss in stress at any location along a tendon depends on the current value and duration of the stress at that location.

- Shrinkage in concrete: A significant cause of prestressing loss is shrinkage shortening of the concrete which houses the tendon. This results in a corresponding shortening of the tendon and thus a direct reduction in tendon stress. The reduced tendon stress slightly reduces the rate and amount of stress loss due to relaxation of the prestressing steel.
- Creep in concrete: In grouted (bonded) post-tensioning systems, there is strain compatibility between the tendon and the concrete. Creep strain in the concrete adjacent to the tendon thus causes a decrease in tendon stress. For unbonded tendons, the decrease in stress along the tendons due to creep of the concrete is generally a function of the overall (average) precompression of the concrete member.

Change in Stress Due to Bending of the Member under Applied Loading

As with nonprestressed steel, the bending of a concrete member due to applied (dead and live) loading results in a change in stress of the prestressing tendons. The stress change for grouted tendons is the same as that of any nonprestressed steel located at the tendon position. The stress change due to bending is usually not viewed as a stress loss since, in most cases, there is an increase in stress. For a rigorous evaluation of the affected member, however, this change in stress must be accounted for, particularly when large deflections are anticipated.

TENDON MODELING

For structural analysis, the prestressing tendons can be modeled either as a loading applied to the hosting member or as a component which resists the applied loading in conjunction with the hosting member. Techniques which model the tendon as applied loading include load balancing, modeling through primary moments, and equivalent load through discretization of the tendon force.

Modeling of Tendon as Applied Loading

Simple Load Balancing

Load balancing, the method introduced by T. Y. Lin [1981] is the simplest and most expedient method of modeling tendons. It is the most commonly used method in building design and when it is applied judiciously and its limitations are recognized, it is a powerful technique.

In its simplest form, load balancing can be applied under the following conditions:

- The member is prismatic with no change in the position of its centroidal axis.
- The tendon profile in each span can be approximated as a single, continuous parabola.
- The change in stress along the length of the tendon is small and does not affect the analysis. In other words, an effective (average) force can be assumed for the tendon.
- The effect of axial loading due to prestressing and the flexure of the member due to prestressing are independent from one another (decoupled).

If the given conditions are met, the impact of a tendon, removed from its housing, can be approximated by uniform loads on each span, as illustrated in the example of Fig. 2.

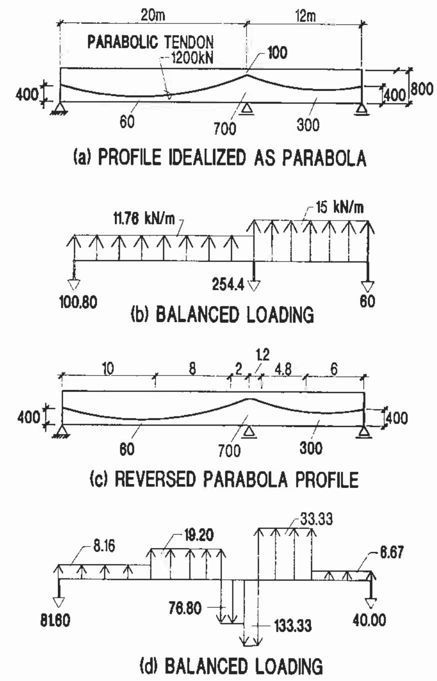


FIG. 2. Simple Load Balancing

For each span of length L the tendon's upward force (w_b) is given by:

$$w_b = 8Pa/L^2$$

where P is the force in the tendon and a is the tendon's drape (the difference between the high and low points).

For the left span of Fig. 2, the drape is:

$$a = 0.5(400+700) - 60 = 490 \text{ mm}$$

The upward force is thus:

$$w_b = 8*1200*0.490/20^2 = 11.76 \text{ kN/m}$$

The force of the tendon on the concrete is considered to balance (offset) a portion of the load on the member, hence the "load balancing" terminology. The loading from the removed tendon (Fig. 2-a) is in self-equilibrium with the reactions it causes at the end of each tendon parabola. The uniform tendon force and its associated concentrated loads are collectively referred to as the balanced loading. The balanced loading is independent of the support conditions of the structural member. Additional information on load balancing is provided in [Aalami, 1990].

In practice, tendons can not be placed with sharp angles over the supports as shown in 2.1.1-1 (a). A gradual curvature, as illustrated in 2.1.1-1(c), is the common selection. Tendon low points are at pre-determined locations (mid-span in most building construction) and there is a gradual reversed curvature over the supports. The force imparted by a tendon to the concrete thus becomes more complex and less amenable to hand calculation. This refinement in simple load balancing is used primarily in association with automated (computer) analysis.

The principal shortcoming of the simple load balancing modeling procedure is that it does not apply to members whose centroidal axis changes along their length, such as members with differences in thickness, or steps. The other shortcoming is that the immediate and long-term stress losses in prestressing must be approximated and accounted for separately.

Extended Load Balancice

Consider the two-span beam shown in Fig. 3. Because the two spans are different depths, the tendon ends on either end of the beam are not aligned. In order to decouple the axial and flexural actions, in accordance with the load balancing concept, it is necessary to add a moment at the point at which the centroidal axis shifts i.e. over the central support. This is shown in 2.1.2-1(b) and is described in more detail in [Aalami, 1990]. Since this approach assumes a constant effective force, the added moment is simply 1200*0.15 = 180kNm where 0.15m is the distance between the centroids of the two spans.

The principal advantage of the extended load balancing approach is its ability to account for nonprismatic members. It does not include a calculation of the prestressing losses.

Tendon Modeling Through Primary Moments

The primary moment, Mp, due to the prestressing force, P, at any location along a member is defined as the prestressing force times its eccentricity, e.

$$Mp = P * e$$

The eccentricity of the force is the distance between the resultant of the tendon force and the centroid of the member. For the example shown in Fig. 4, the moment at the right end of the first span is:

$$Mp = 1200\,(400\text{-}100)/1000 = 360\text{ kNm}$$

The primary moment can be used as an applied loading in lieu of the balanced loading for structural analysis. This modeling technique is more commonly used by bridge designers than building designers. It has the advantage of implicitly accounting for nonprismatic sections – a condition which is common in bridge construction. An added advantages is that by considering the primary moment at each section to be the duly adjusted

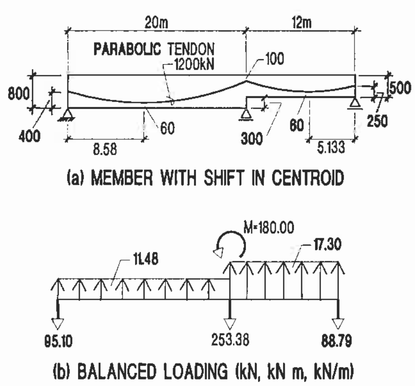


FIG. 3. Extended Load Balancing (Note: Value In mm Unless Noted Otherwise)

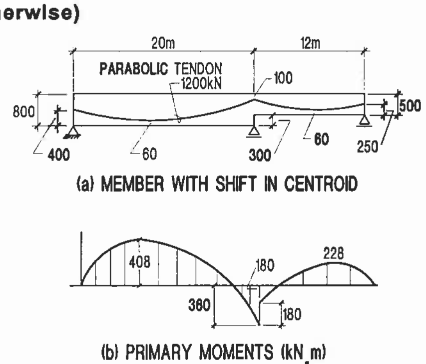


FIG. 4. Modeling through Primary Moments

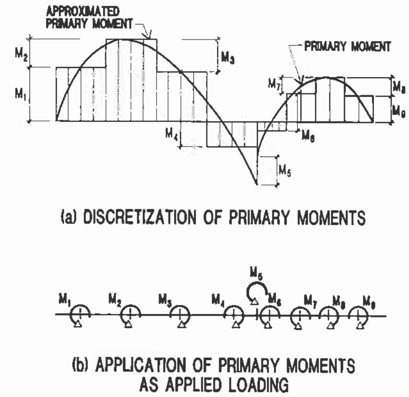


FIG. 5. Discretization and Application of Primary Moments as Loading

force at that section times the eccentricity at the section, prestress losses along the tendon can be included. Note that if this option is adopted, the axial component of the prestress loading must be represented in its variable form to maintain an equilibrium of forces.

The primary moment, whether or not is adjusted for prestress losses, depends only on the force in the tendon, the tendon profile and the cross-sectional geometry of member. It is independent of the number and location of member supports or the support conditions.

In practice, the primary moment diagram is discretized into a number of steps as illustrated schematically in Fig. 5. Each discrete moment shown in part (b) of the figure is equal to the change in the value of moment between two adjacent steps in the primary moment diagram. Note that the change of moment due to the shift in the centroidal axis (M_5) is automatically accounted for.

For the example shown

$$M_1 + M_2 = 408\text{ kNm}$$

Equivalent Load Through Discretization of Tendon Force

In this modeling scheme, the tendon force is discretized along its length to achieve the following improvements in accuracy:

- The method accounts for the variation of force along the tendon length caused by the friction and seating of the tendon at stressing.
- The method accounts for losses in prestressing due to creep and shrinkage using an approximate procedure.

Consider a span of a continuous member and its prestressing as shown in Fig. 6. The tendon length is idealized as a series of straight line segments, typically 20 segments per span. Fig. 7 shows a portion of the discretized tendon. The actual distribution of prestressing force is the smooth curve marked "actual" in part (b) of the figure. For a tendon idealized as a series of straight segments there is a gradual stress loss along each segment due to wobble friction. The component of friction due to curvature (angle change), however, is concentrated at the intersection of the segments (marked as node i, i+1, ...). Hence the force distribution would be represented by a series of sloping lines with steps at the discretization points as shown in part (b). The force distribution can be further simplified by considering the force in each tendon segment to be equal to the force at the mid-point of the segments, as shown in part (c).

For a representative tendon node i, the tendon forces of the

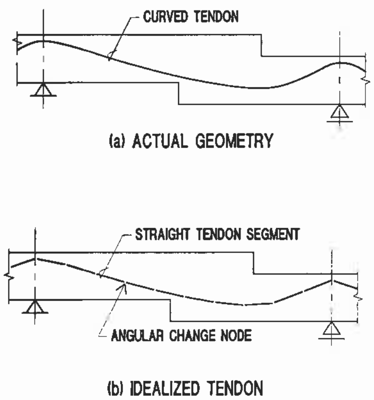


FIG. 6. Tendon Presentation by Discretization

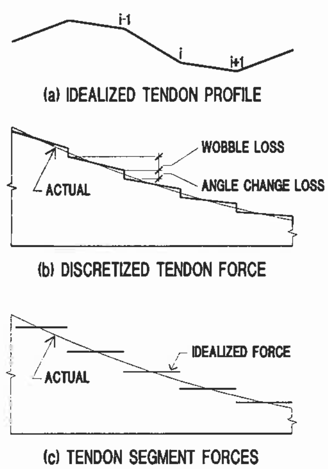


FIG. 7. Idealization of Tendon Force

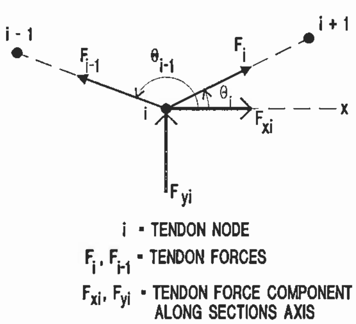


FIG. 8. Resolution of Tendon Actions at Discretization Points

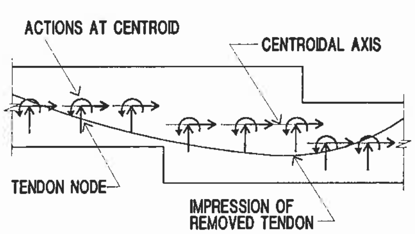


FIG. 9. Equivalent Tendon Actions at Centroidal Axes

adjacent segments are F_i and F_{i-1}, as shown in Fig. 8. The two segment forces can be resolved into equivalent forces F_{xi} and F_{yi}, parallel and perpendicular to the centroidal axis of the hosting member. These two force components can be transferred to the centroid of the section with the addition of a moment, M_i, equal to $F_{xi}*e$, where e is the eccentricity of the tendon at tendon node i. Fig. 9 illustrates a series of equivalent tendon actions placed at the member centroid using this method.

This scheme, an iterative solution strategy which involves the coupling of the tendon force and its equivalent loading, results in an analysis in which immediate prestress losses can be rigorously accounted for and long term losses can be approximated. At each iteration the prestress losses are computed on the basis of the current prestressing force. The current force is then used to compute the equivalent forces, which will, in turn, change the long-term losses. The iteration is continued until convergence is achieved [ADAPT-PT, 1999].

Modeling of Tendon as a Load Resisting Element

Unlike the modeling schemes described in the previous sections, when the tendon is modeled as a load resisting element, it is not considered as removed from the concrete member. Rather, the tendon is modeled as a distinct element linked to the concrete member. One other characteristic of this approach is that the allowance for long-term stress loss becomes an implicit feature of the computations. Separate stress loss computations are not required.

Fig. 10, which shows a partial elevation of a [illegible] is used to illustrate the modeling. For simplicity, only one prestressing tendon is shown. Before describing the discrete tendon modeling option however, the equivalent load tendon modeling is reviewed.

Parts (b), (d) and (f) of the figure illustrate the representation of a tendon by means of equivalent loading. Note that the tendon is considered removed from its housing. The initial forces imparted by the tendon to the segment are transferred to the centroid of the segment (part d). These forces are considered as a constant applied loading which is not affected by creep, shrinkage, or deformation of the segment. Long-term loss effects are accounted for at a later stage.

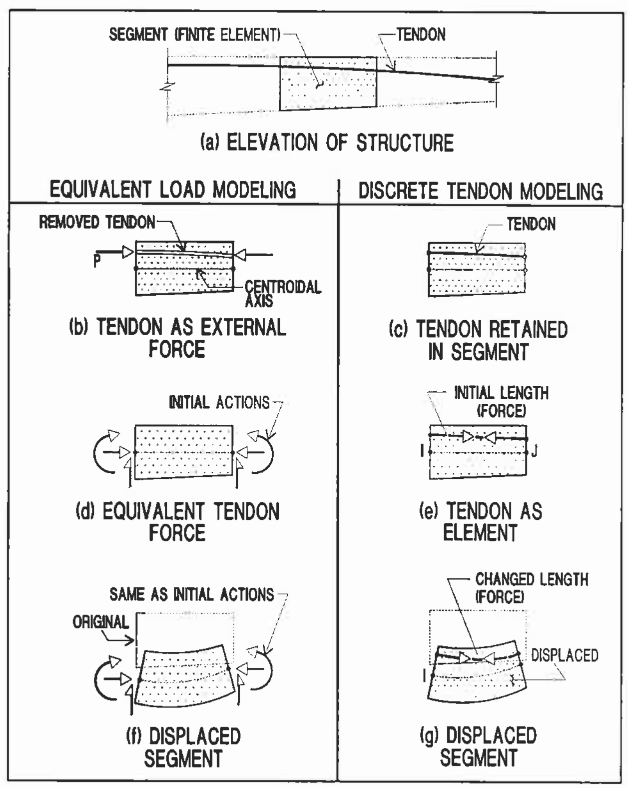
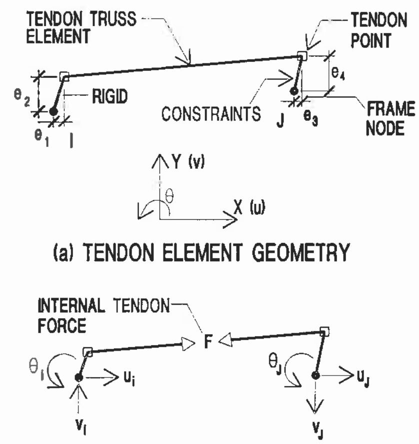


FIG. 10. Equivalent Load and Discrete Modelings

In contrast, in the discrete tendon modeling scheme, the tendon is retained in position (Fig. 10-c). Each tendon in the segment is viewed as an independent element, subject to displacement and change in stress based on the deformation of the segment within which it is housed, or to which it is locked (external tendons). Each tendon element is assumed to have an initial force which is determined from friction loss calculations. Any subsequent deformation of the concrete segment, such as shown in (g), will be accompanied by a compatible displacement of the tendon element, using the requirement of plane sections remaining plane (Fig. 11). The displacement of the tendon ends at the faces of the hosting concrete segment results in a change in tendon force.

Observe that in this modeling scheme there is an implicit interaction between the deformation of the hosting concrete segment and the force in tendon, irrespective of the cause of the deformation. As a result, it is not necessary to calculate the deformations due to creep and shrinkage separately in order to modify the tendon force. Likewise, the change in prestressing force due to relaxation is automatically accounted for in the equilibrium equations set up for the analysis of the segment.

This modeling scheme is the core of analysis software such as ADAPT-ABI [1999]. A similar modeling scheme can be applied to three dimensional problems such as floor slabs and bridge decks where the constituent elements of the slab can be viewed as consisting of concrete plate elements with embedded prestressing tendon elements (Fig. 12).



(a) TENDON ELEMENT GEOMETRY

(b) TENDON ELEMENT DEGREES OF FREEDOM

FIG. 11. Prestressing Tendon Idealization

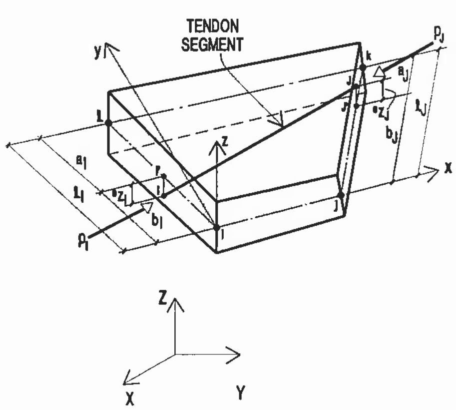


FIG. 12. Geometry of Prestressing Tendon Segment Passing through Finite Element

HYPERSTATIC (SECONDARY) ACTIONS

The structural modeling and analysis of prestressed members is not complete unless the hyperstatic (secondary) actions caused by prestressing are considered. Hyperstatic actions are the forces and moments generated in the member as a result of constraint by the member's supports to free deformation of the member. Consider an extension of the example of Fig. 3 in Fig. 13. When the balanced loading shown in part (b) of the figure is applied to the structure, the supports constrain the deformation of the structure. As a result, the forces at the supports shown in part (a) are generated. These reactions are referred to as hyperstatic (secondary) actions. They are in self-equilibrium, and are in response to the application of the balanced loading, which is also in self-equilibrium. The hyperstatic reactions cause a distribution of moment in the structure (hyperstatic moments) as shown in part (d).

As the terminology implies, hyperstatic moments are characteristic of indeterminate structures. The application of balanced loading to a structure results in prestressing (post-tensioning) moments. In an indeterminate structure, the prestressing (post-tensioning) moments (M_{pre}) are the sum of the primary (M_p) and hyperstatic (M_{hyp}) moments.

$$M_{pre} = M_p + M_{hyp} \qquad (2)$$

For example, for the right end of span 1:

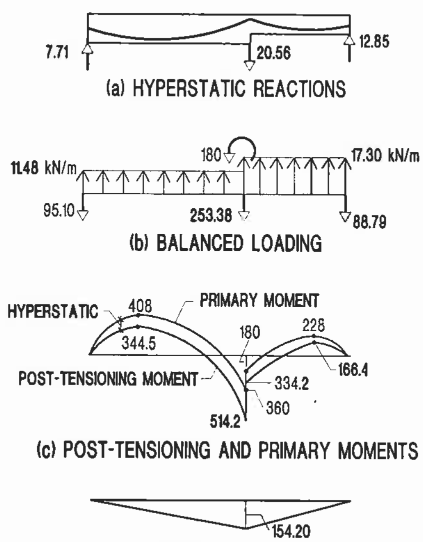
M_{pre} = 514.20 kNm; M_p = 360 kNm ; M_{hyp} = 154.2 kNm

The computation of hyperstatic moments is important, since for the design of a section it is the value of the hyperstatic moment which is contributory to the total moment demand.

NUMERICAL EXAMPLE

This example is intended to illustrate the application of discrete modeling of tendons to the example used in the previous section.

- Beam dimensions
  Span 1----500x800 mm
  Span 2----500x500 mm
- Concrete strength----40 MPa



(a) HYPERSTATIC REACTIONS

(b) BALANCED LOADING

(c) POST-TENSIONING AND PRIMARY MOMENTS

(d) HYPERSTATIC (SECONDARY) MOMENT

FIG. 13. Prestressing Actions

TABLE 1. Tendon Modeling Schemes and Features

ɔdeling method (1)	Flexure/ membrane Stresses (2)	Solution mode (3)	Losses at stressing included (4)	Allows nonprismatic sections (5)	Long-term losses included (6)	Tendon modeling (7)	Member deformation included (8)
ncing	De-coupled	Effective force	No	No	No	Applied load	No
alancing	De-coupled	Effective force	No	Yes	No	Applied load	No
;h primary moments	De-coupled	Effective force	No	Yes	No	Applied load	No
ivalent load	Coupled	Variable force	Yes	Yes	No	Applied load	No
idealization	Coupled	Variable force	Yes	Yes	Yes	Resisting element	Yes

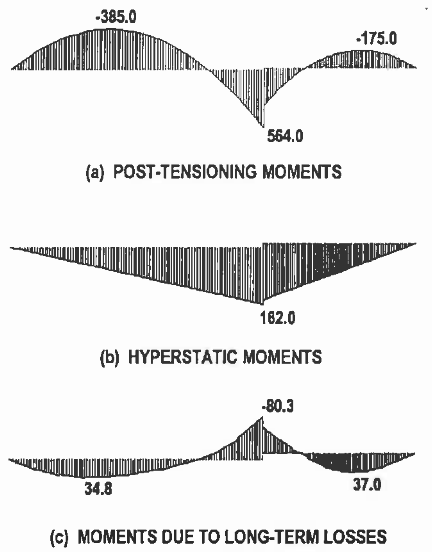


FIG. 14. Posttensioning Actions and Losses Using Discrete Tendon Modeling (kN-m)

- Concrete long-term model----ACI 209-78
- Ultimate creep coefficient----2.5
- Ultimate shrinkage coefficient----0.000400
- Bonded Tendon
  Tendon area----988 mm^2
  Coefficient of angular friction----0.25 /radian
  Coefficient of wobble friction----0.000066 /m
  Jacking force (left end)----1470 kN
  Applied loading: dead load only----12 kN/m

Using a discrete tendon modeling software [ADAPT-ABI, 1999], a solution for the deformation and stresses after 20 years was obtained. Each span was modeled as 10 segments. The post-tensioning moments, hyperstatic moments and the moments due to creep and shrinkage of the beam are shown in Fig. 14. As can be seen, the change in post-tensioning moment in the second span due to creep and shrinkage is an increase from 514.20 to 564.00 kNm. The hyperstatic moment over the interior support (162 kNm) is about 30% of the post-tensioning moment (564 kNm).

The drop in tendon force after twenty years relative to the jacking force is illustrated in Fig. 15.

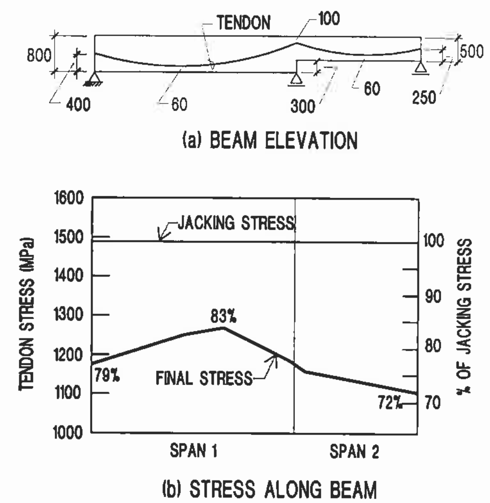


FIG. 15. Stress Loss in Tendon after 20 Years

CONCLUDING REMARKS

This state-of-the-art work offers a clear perspective of the structural models used in the analysis of post-tensioned concrete members. The limitations and merits of each modeling scheme are described and illustrated through numerical examples.

Table 5-1 provides an overview of the features of each of the modeling schemes. Improvements in computational techniques and computing power are now allowing the traditional, approximate, long-term loss calculations to be replaced by techniques such as discrete tendon modeling which include implicit calculation of long-term losses. Although this paper has discussed discrete tendon modeling in conjunction with post-tensioned members, similar benefits are obtained when the technique is applied to pre-tensioned members.

APPENDIX. REFERENCES

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Comment

Page 158, second column: expression given to arrive at 11.76 kN/m is approximation used in engineering offices. The values given in the figures and used in the computations presented are based on the equations of parabola. They may differ from the approximated formula.